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Thermodynamics of a Quantum Simulating Fluid Complex

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Abstract

Earlier work by the present author on a two fluid quantum simulating system is elaborated and clarified. Both of the fluids involved are assumed to have positive and negative mass in their structure. The thermodynamical and statistical consequences of using such a mixed mass system are analysed. Connections with the work of other authors are established. In particular, bulk viscosity is now incorporated as a vital characteristic of one of the fluids.

1. *Introduction*

This paper is concerned with the elaboration, extension and clarification of earlier work (Gilson, 1969a, b, c, 1971a) by the present author and the employment of an idea contained in the work (Hörnqvist, 1969). We shall, to a large extent, be concerned with showing how a two-fluid reformulation of Schrödinger quantum mechanics on to a 'classical' statistical fluid basis can be linked unambiguously to standard thermodynamical and fluid concepts.

If one accepts the need to advance beyond the now orthodox quantum mechanical concepts characterised by Schrödinger theory and such imponderables as wave functions (which have little, if any, physical pictorial quality as against their obvious and great mathematical utility); then, as has been suggested by Bohm (1957), it seems that we must work with classical like fluids with 'extra' degrees of freedom. The present author (Gilson, 1969c) has shown that use can be made of negative mass in resolving the problems which arise in this context. Thus, in the present work, we shall assume that quantum phenomena are basically due to the interaction of two distinguishable fluids. We shall call these two fluids the solute and the solvent. Considerable evidence for the validity of this decomposition has been given by the present author in earlier work. Further, it seems inevitable that both of these fluids will have positive and negative mass involved in their structures. This inevitability will be discussed further in a later section. The first problem which arises when working with mixed mass distributions, in thermal velocity space, concerns averages. How should averages be defined when negative mass regions occur in thermal velocity space? The answer to this question is given in the next section.

2. *Averages with Negative Mass*

Let us for the moment consider the solute part of the fluid complex. As elsewhere (Gilson, 1969b), this fluid will be denoted by the subscript '1' and itself is to be regarded as decomposable into positive and negative mass constituents. We shall have in the scheme a basic 'molecular' mass. This will be the rest mass of an electron and will be denoted by m_e. The positive mass constituent particles of the solute will be assumed to have rest mass

$$m_{1+} = m_e \tag{2.1}$$

and the negative mass constituent particles of the solute will be assumed to have rest mass

$$m_{1-} = -m_e \tag{2.2}$$

The concentrations (molecules per cc) of these two constituents of the solute will be denoted by n_{1+} and n_{1-}. Thus the positive and negative mass densities will be

$$\rho_{1+} = n_{1+} m_{1+} \qquad \text{and} \qquad \rho_{1-} = n_{1-} m_{1-} \tag{2.3}$$

The total mass density for the solute will be

$$\rho_1 = n_{1+} m_{1+} + n_{1-} m_{1-} \tag{2.4}$$

We shall be particularly concerned with how the mass distribution in configuration space given by (2.4) is built up from a 'thermal velocity space' mass density distribution. The thermal velocities [or peculiar velocities (Chapman, 1960)] for the solute will be denoted by

$$\mathbf{l}_{1\pm}(\mathbf{r}, t) = \mathbf{v}_{1\pm}(\mathbf{r}, t) - \langle \mathbf{v}_1(\mathbf{r}, t) \rangle \tag{2.5}$$

where $\langle \mathbf{v}_1(\mathbf{r}, t) \rangle$ is the mass-weighted mean solute velocity at position $\mathbf{r}$ and time t. In fluid systems, it is usual to define two mean velocities, $\bar{\mathbf{v}}_A$ and $\langle \mathbf{v}_1 \rangle$, by equations such as

$$(n_{1+} + n_{1-}) \bar{\mathbf{v}}_A = n_{1+} \bar{\mathbf{v}}_{1+} + n_{1-} \bar{\mathbf{v}}_{1-} \tag{2.6}$$

and

$$(\rho_{1+} + \rho_{1-}) \langle \mathbf{v}_1 \rangle = \rho_{1+} \bar{\mathbf{v}}_{1+} + \rho_{1-} \bar{\mathbf{v}}_{1-} \tag{2.7}$$

Equation (2.6) gives the number average and (2.7) gives the mass average. In Chapman and Cowling's book, the subscript '0' is used to denote the mass average. It is the mass weighted average which plays the important part in this work. When negative contributions do occur in such a weighted average, they will clearly have a most significant effect on the characteristics of the system and involve possibilities which could not otherwise occur. From (2.3) and (2.7), we have

$$m_e(n_{1+} - n_{1-}) \langle \mathbf{v}_1 \rangle = m_e(n_{1+} \bar{\mathbf{v}}_{1+} - n_{1-} \bar{\mathbf{v}}_{1-}) \tag{2.8}$$

or

$$\langle \mathbf{v}_1 \rangle = N_1^{-1}(n_{1+}\bar{\mathbf{v}}_{1+} - n_{1-}\bar{\mathbf{v}}_{1-}) \tag{2.9}$$

where

$$N_1 = n_{1+} - n_{1-}. \tag{2.10}$$

We now assume that

$$n_{1+} = \int f_{1+}(\mathbf{l}_{1+}, \mathbf{r}, t)\, d\mathbf{l}_{1+} \tag{2.11}$$

and

$$n_{1-} = \int f_{1-}(\mathbf{l}_{1-}, \mathbf{r}, t)\, d\mathbf{l}_{1-} \tag{2.12}$$

where f_{1+} and f_{1-} are the thermal velocity space distribution densities and $d\mathbf{l}_{1\pm}$ are elements of volume in thermal velocity space. Thus (2.9) becomes

$$\langle \mathbf{v}_1 \rangle = N_1^{-1} \int (f_{1+} - f_{1-})\, \mathbf{v}_1\, d\mathbf{l}_1 \tag{2.13}$$

This is the type of average we shall use. It is convenient to define an unnormalised mass density in velocity space, $m_1(l_1)$, by

$$\tilde{m}_1(l_1) = (f_{1+} - f_{1-})\, m_e \tag{2.14}$$

and a related normalised mass density by

$$m_1(l_1) = N_1^{-1}(f_{1+} - f_{1-})\, m_e \tag{2.15}$$

We see from (2.10), (2.11) and (2.12) that

$$\int m_1(\mathbf{l}_1)\, d\mathbf{l}_1 = m_e \tag{2.16}$$

Thus the normalisation of the solute mass density is to the value of one electron mass. Generally, we can define a solute mass weighted average for functions of l_1 as

$$m_e \langle F \rangle_1 = \int m_1(\mathbf{l}_1)\, F(\mathbf{l}_1)\, d\mathbf{l}_1 \tag{2.17}$$

Let us now turn our attention to the solvent. Almost identical considerations apply, in forming a mixed mass distribution for this fluid, as were applicable for the solute. However, there are some differences. For one thing, we shall define the normalisation number in this case by

$$N_2 = n_{2-} - n_{2+} \tag{2.18}$$

This number is greater than zero provided that n_{2-} is the dominant concentration. We shall, in fact, make $m_2(\mathbf{l}_2)$ a dominantly negative mass distribution. Thus the unnormalised distribution

$$\tilde{m}_2(\mathbf{l}_2) = (f_{2+} - f_{2-})\, m_e \tag{2.19}$$

and the normalised distribution

$$m_2(\mathbf{l}_2) = N_2^{-1}\, \tilde{m}_2(\mathbf{l}_2) \tag{2.20}$$

will have the same sign. They will, in fact, both be negative if f_{2-} is greater than f_{2+}. From (2.20), it follows that

$$\int m_2(\mathbf{l}_2)\,d\mathbf{l}_2 = -m_e \tag{2.21}$$

Thus the solvent mass distribution, $m_2(\mathbf{l}_2)$, is normalised to minus the mass of one electron. The reader should be warned that the notation here is not the same as that used in (Gilson 1969b, c, 1971a) because we are now taking the physical solvent mass density to be dominantly negative. That is to say, our old solvent mass density multiplied by minus unity is now our new solvent mass density. This has the effect that there is a change of sign for all solvent averages. The final results and conclusions are not altered, however, because now we shall add solute and solvent energy densities whereas before we subtracted solvent energy density from solute energy density. Our present definitions and notations are nearer to the physical meaning of the terms and so are preferable to our earlier forms. Generally, we shall have

$$m_e\langle F\rangle_2 = \int m_2(\mathbf{l}_2)\,F(\mathbf{l}_2)\,d\mathbf{l}_2 \tag{2.22}$$

The normalisations (2.16) and (2.21) are easily converted to plus or minus unity respectively by dividing the normalised distributions (2.15) and (2.19) by the electron rest mass, m_e. The actual form to be taken by distributions such as $m_1(\mathbf{l}_1)$ and $m_2(\mathbf{l}_2)$ in order that they will generate quantum mechanics is known from the work (Gilson, 1971a, equations 4.1 and 4.4). They are closely related to Maxwell distribution but because of their local character and divergence problems they are mathematically quite difficult to handle. However, we shall not need to know their actual form of dependence on T and $\mathbf{l}$ for the work in this paper. The temperature of the solute can now be defined by

$$\tfrac{3}{2}kT_1 = \tfrac{1}{2}\langle m_e \mathbf{l}_1{}^2\rangle \tag{2.23}$$

and the temperature of the solvent by

$$\tfrac{3}{2}kT_2 = \tfrac{1}{2}\langle m_e \mathbf{l}_2{}^2\rangle \tag{2.24}$$

We shall now briefly consider why the four-fold decomposition denoted by '± 1' and '± 2' is a necessary feature of our scheme.

3. *Why Four States*

The real part of the one-particle Schrödinger equation has the form

$$E_Q = \mathrm{Re}\left(i\hbar\frac{\partial \ln\psi}{\partial t}\right) = \frac{m_e}{2}\langle \mathbf{v}_1\rangle^2 - \frac{m_e}{2}\langle \mathbf{v}_2\rangle^2 + 2\mu + V \tag{3.1}$$

where

$$m_e\langle \mathbf{v}_1 - i v_2\rangle = i\hbar\nabla\ln\psi \tag{3.2}$$

and V is the external potential. One of the μ's in equation (3.1) arises from the solute fluid and the other μ arises from the solvent fluid and they are equal to each other. The μ's are thermal energies associated with the 'peculiar' velocities relative to the respective fluids mean velocity (Gilson, 1969b). Thus, a fluid simulation for equation (3.1) must involve the average kinetic energy of the solute being positive going with a positive sign for μ, and the average kinetic energy of the solvent being negative also going with a positive sign for μ. It would seem to be impossible for the solvent to involve negative energy particles only and yet (when $\mu > 0$) to make a positive contribution to the thermal energy as a result of their 'peculiar' motion distribution. However, if the solvent has positive energy constituents as well as the dominantly negative energy particles in its structure, then its translational kinetic energy, $-m_e\langle\bar{\mathbf{v}}_2\rangle^2/2$, can, as indicated, be negative; whilst its thermal energy, $\mu_2 = \mu_{2+} + \mu_{2-}$ can be positive as a result of, μ_{2+}, the positive particle contribution to the solvent thermal energy, being numerically greater than, μ_{2-}, the negative particle contribution to the solvent thermal energy. The reader is reminded that there has been a change of notation from what was used in earlier papers. In particular μ_2 here corresponds to $-\mu$ (solvent) in (Gilson, 1969b, c).

As we have mentioned elsewhere (Gilson, 1971a), the solute and solvent can interchange their roles where the temperature, $T(x, t)$, changes sign. Thus there is a general necessity for both solute and solvent to involve both signs of mass and so the four states can be seen to be essential.

4. *Internal and External Functions*

Basic to the philosophy of this work is the idea that Schrödinger quantum mechanics arises from the fact that in classical relativity there is a place for negative mass. This connection has only shown up as a result of attempting to derive the Schrödinger equation from a statistical fluid basis (Gilson, 1969c). This relativistic fluid route to quantum mechanics also suggests another feature of structure which will be useful to us. It suggests that there occur, in the positive negative mass mixture context, two types of function which we shall call 'internal' and 'external'. This can be seen by considering the two relativistic equations (4.1) and (4.2) for particles of rest mass, $\pm m_e$, in scalar potentials V_1 and V_2.

$$\left(\frac{E_1 - V_1}{c}\right)^2 = p_1{}^2 + (m_e c)^2 \tag{4.1}$$

$$\left(\frac{E_2 - V_2}{c}\right)^2 = p_2{}^2 + (m_e c)^2 \tag{4.2}$$

If we define

$$E_1 + E_2 - (V_1 + V_2) = E_{\text{int}} - W_{\text{int}} \tag{4.3}$$

and

$$E_1 - E_2 - (V_1 - V_2) = E_{\text{ext}} = 2m_e c^2 \tag{4.4}$$

with

$$V_1 + V_2 = W_{\text{int}} \tag{4.5}$$

and

$$V_1 - V_2 = W_{\text{ext}} \tag{4.6}$$

then

$$E_{\text{int}} = \frac{p_1{}^2 - p_2{}^2}{2m_e} + W_{\text{int}} \tag{4.7}$$

Expression (4.7) for E_{int} can be seen to be comparable with the Schrödinger equation (3.1). On the other hand, expression (4.4) for E_{ext} gives twice the rest energy for the particle involved in E_{int}. Thus for a system which is closed in the sense that $V_1 - V_2 = W_{\text{ext}} = 0$ (that is the particles are only interacting with each other), the rest mass appears as an external energy function and the classical like kinetic energy appears as an internal energy function. The reader may prefer to see the argument the other way. That is with $V_1 + V_2 = 0$. However, he will still arrive at an equation like (4.7) and similar conclusions. The reason for this duality effect seems to be that the use of negative mass in systems widens the whole concept of what is meant by an 'open' or a 'closed' system. This duality also led to this author making the wrong choice in defining the sign for mass averages involving the solvent in earlier work. The preceding argument is only meant to be suggestive and, indeed, if an equation such as (4.7) is to be linked with the Schrödinger equation a more involved form is needed (Gilson, 1969c). However, we have given a motivation for recognising the two types, 'internal' and 'external' functions. In particular, the 'classical energy' and the 'rest mass' appear on opposite sides of the fence, as it were.

5. *Equations of Motion and Constraint*

The equation of continuity for the solute mass density, ρ_1, can be taken to be

$$\frac{\partial \rho_1}{\partial t} = -\nabla . (\langle \mathbf{v}_1 \rangle \rho_1) \tag{5.1}$$

Here

$$\rho_1 = m_e N_1 \tag{5.2}$$

Here we shall now add an important additional concept to this author's previous work. We shall assume that the solvent is a viscous fluid. This idea comes from the work of Nils Hörnqvist (1969) where it is used in the one fluid context. With Hörnqvist we shall regard this viscosity as a 'bulk' viscosity effect and in the present work no other viscosity will be brought into the structure. Thus we can take the stress tensor for the 'bulk' viscous solvent to be

$$\sigma_{2i\kappa} = -p_2 \delta_{i\kappa} - \nu \rho_2 \delta_{i\kappa} \nabla . \langle \mathbf{v}_2 \rangle \tag{5.3}$$

$$= \sigma_2 \delta_{i\kappa} \tag{5.4}$$

say, where ν is the coefficient of kinematic bulk viscosity. On the other hand, for the non-viscous solute the stress tensor will be

$$\sigma_{1i\kappa} = -p_1\,\delta_{i\kappa} = \sigma_1\,\delta_{i\kappa} \tag{5.5}$$

say. For the viscous solvent we shall also make use of the first approximation transport equation

$$\langle \mathbf{v}_2 \rangle \rho_2 = +D\nabla\rho_2 \tag{5.6}$$

Here,

$$\rho_2 = -m_e N_2 \tag{5.7}$$

The coefficient of diffusion, D, will be taken to have the value

$$D = \hbar/2m_e \tag{5.8}$$

which is the same numerical identification as was made by Nelson (1966, 1967). The temperatures for the two fluids were defined by equations (2.23) and (2.24).

The osmotic pressure of the solute plays an important role and will be denoted by P_0. It is defined by

$$P_0 = p_1 - p_2 = \tfrac{1}{3}\int \mathbf{l}_1{}^2\,\tilde{m}_1(\mathbf{l}_1)\,d\mathbf{l}_1 \tag{5.9}$$

where p_2 is the hydrostatic pressure due to the solvent. We are now in a position to state the equilibrium requirements for the fluid complex. Solute and solvent will be assumed to be locally in mutual thermal equilibrium and consequently they will have the same temperature at any space point and definite time. Thus

$$T_1(\mathbf{r}, t) = T_2(\mathbf{r}, t) \tag{5.10}$$

The temperature will, as indicated, be assumed to vary with space and time. The second equilibrium requirement is that the solute stress forces exactly balance the stress forces due to the motion of the viscous solvent. This condition is expressed by

$$\sigma_{1i\kappa} = \sigma_{2i\kappa} \tag{5.11}$$

From (2.14), (2.15), (2.23), (5.2) and (5.9), we get

$$\frac{p_1 - p_2}{\rho_1} = \frac{kT}{m_e} \tag{5.12}$$

Thus the osmotic pressure for the solute satisfies the perfect gas equation of state. From (5.3), (5.4) and (5.5) we get

$$\sigma_1 = -p_1 = \sigma_2 \tag{5.13}$$

or

$$p_1 - p_2 = -(\sigma_2 + p_2) \tag{5.14}$$

Hence from (5.3), (5.12) and (5.14) we deduce that

$$\frac{p_1 - p_2}{\rho_1} = -\frac{(\sigma_2 + p_2)}{\rho_1} = +\nu\frac{\rho_2}{\rho_1}\nabla.\langle\mathbf{v}_2\rangle$$
$$= kT/m_e \tag{5.15}$$

Our third and final equilibrium requirement will be to take the total local mass density ρ_T to be zero at all points of configuration space. Thus

$$\rho_T = \rho_1 + \rho_2 = m_e(N_1 - N_2) \equiv 0 \tag{5.16}$$

or

$$\rho_2/\rho_1 = -1 \tag{5.17}$$

The three equilibrium conditions are probably not independent. It is likely that they are all deducible from a single principle. This point will be left for further study. We remark that, if ρ_T is identified as an external function, then the condition (5.16) means that the total external fluid mass density is zero everywhere and at all times. Expressed otherwise; the mass distribution associated with the fluid complex is not seen externally. On the other hand,

$$\rho_0 = \rho_1 - \rho_2 = m_e(N_1 + N_2) \tag{5.18}$$

will be the corresponding internal mass density function and it turns out that ρ_0/m_e is the usual quantum probability density.

Equations (5.6), (5.15) and (5.17) give the correct form for the local temperature, $T(\mathbf{r}, t)$:

$$\frac{kT}{m_e} = -r\nu D\nabla^2 \ln \rho_2 \tag{5.19}$$

Elsewhere, we have obtained (Gilson, 1971a, equations 2.6, 3.4, 6.5) an expression for the local temperature in the one-dimensional case. We need only replace $\partial^2/\partial x^2$ in this way by ∇^2 to get the three-dimensional form

$$\frac{3kT}{m_e} = -\frac{\hbar^2}{4m_e^2}\nabla^2 \ln \rho \tag{5.20}$$

where $\rho = \psi^* \psi$. Comparing (5.19) and (5.20), we see that, ρ_2/m_e can be identified as some constant times ρ,

$$\rho_2 = -c\rho m_e \tag{5.21}$$

and because of (5.17) we then have

$$\rho_1 = +c\rho m_e \tag{5.22}$$

Actually, we do not need to appeal to results obtained elsewhere to get (5.21) and (5.22). We can alternatively assume that $\rho = \psi^* \psi$, as employed in conventional quantum mechanics is the total internal function, $(\rho_1 - \rho_2)/m_e$, in our scheme. Thus

$$\rho m_e = \rho_1 - \rho_2 \tag{5.23}$$

taken with (5.17) gives

$$\rho_1 = \tfrac{1}{2}\rho m_e \qquad \text{and} \qquad \rho_2 = -\tfrac{1}{2}\rho m_e \tag{5.24}$$

That is with this assumption c has the explicit value 1/2. We can from (5.19) and (5.20) also identify

$$3\nu D = \frac{\hbar^2}{4{m_e}^2} \tag{5.25}$$

Using the value (5.8) in (5.25) gives

$$\nu = \frac{\hbar}{6m_e} \tag{5.26}$$

Thus from (5.8) and (5.26) we have

$$(D/\nu) = 3 \tag{5.27}$$

This is a very interesting relationship, probably of deep significance. We also note that the values of the two thermal energies μ_1 and μ_2 are from (2.23) and (2.24)

$$\mu_1 = \tfrac{3}{2}kT \qquad \text{and} \qquad \mu_2 = \tfrac{3}{2}kT \tag{5.28}$$

the temperatures both being taken equal to T because of (5.10). Thus for the two specific heats at constant volume per particle, we have

$$C_{v,1} = C_{v,2} = (3/2)k \tag{5.29}$$

From (5.28), it follows that

$$\mu_1 = \mu_2 \tag{5.30}$$

This is equivalent to the result obtained in (Gilson, 1969b) and indicates an equi-partition of thermal energy between the solute and the solvent. Equation (5.30) shows why our present notation is superior to the earlier notation when we had μ (solute) $= -\mu$ (solvent). This was less clear physically than is equation (5.30).

6. *Contributions to the Entropy*

The solute satisfies the perfect gas equation (5.12) relative to the solvent pressure. Thus we can associate an entropy per particle with the solute of the form

$$s_1 = c_v \ln T - k \ln \rho + a \tag{6.1}$$

$$= (3/2)k \ln T - k \ln \rho + a \tag{6.2}$$

by (5.29). If the value of T in terms of ρ (5.20), is substituted into (6.2) we get

$$s_1 = (3/2)k \ln |\nabla^2 \ln \rho| - k \ln \rho + b \tag{6.3}$$

Let us compare our normalised distribution $m_1(\mathbf{l}_1)$ which was given explicitly in (Gilson, 1971a) for one dimension and which in three dimensions will have the form

$$m_1(\mathbf{l}_1) = c_1 |T|^{-3/2} \exp\left(-m_0 \frac{(\mathbf{v}_1 - \langle \mathbf{v}_1 \rangle)^2}{2kT(x, t)}\right) \tag{6.4}$$

where c_1 is a constant, with the type of distribution often used in statistical mechanics (Sommerfeld, 1967)

$$f = N^{-1} e^{-\alpha} \exp(-\epsilon(\mathbf{l})/kT) \tag{6.5}$$

It follows that

$$\frac{e^{-\alpha}}{N} \equiv \frac{\lambda}{|T|^{3/2}} \tag{6.6}$$

where λ is a constant. If z_0 is the partition function, we also have

$$e^{\alpha} = z_0 N^{-1} \equiv \lambda^{-1} |T|^{3/2} N^{-1} \tag{6.7}$$

Hence

$$z_0 = \lambda^{-1} T^{3/2} \tag{6.8}$$

and Boltzmann's form for the entropy per particle is thus

$$s_1 = k \ln z_0 + \mu_1/T \tag{6.9}$$

where μ_1 is given by (5.26). Using (6.8) this reduces to

$$s_1 = (3/2) k \ln T + \mu_1/T + \text{constant} \tag{6.10}$$

The term in $\ln \rho$ in (6.2) does not occur in (6.10) because the distribution $m_1(\mathbf{l}_1)$ has been normalised to a constant value over thermal velocity space and so does not contain direct information about, ρ, the configuration space density. The term μ_1/T in (6.10) was calculated in Gilson (1971a) and as usual for a perfect gas has the constant value $(3k/2)$ per particle in three dimensions. This value is also clear from (5.28). This same term can be included in the constant in (6.2). Thus we shall take the entropy of the solute to be given by

$$s_1 = \tfrac{3}{2} k \ln T - k \ln \rho + \mu_1/T + A_1 \tag{6.11}$$

Similarly we can take the entropy of the solvent to be

$$s_2 = (3/2) k \ln T - k \ln \rho + \mu_2/T + A_2 \tag{6.12}$$

The case for (6.12) is not as good as the case for (6.11) because the solvent is a rather more complex fluid than is the solute. One difficulty with the solvent entropy arose when evaluating the μ_2/T term because the solvent distribution involves for one thing a divergent exponential. However, this divergence is damped by negative mass contributions (positive on our present view) and this difficulty was overcome in Gilson (1971a). Anyway, here we have the unambiguous result (5.30). Thus the internal local entropy of our fluid complex will be taken to be

$$s_{\text{int}} = s_1 + s_2 = 2k\ln(T^{3/2}/\rho) + \frac{\mu_1 + \mu_2}{T} + A_1 + A_2 \qquad (6.13)$$

and the external local entropy can be taken to be

$$s_{\text{ext}} = s_1 - s_2 \equiv 0 \qquad (6.14)$$

if we agree to take $A_1 - A_2 = 0$ or, indeed, take both A_1 and A_2 zero. The external entropy is here zero in agreement with Nernst's theorem that the entropy of a pure quantum state can be taken to be zero at absolute zero temperature. The temperature T used in this paper can be regarded as an internal temperature with $T_{\text{ext}} = T_1 - T_2 \equiv 0$.

7. *Fokker–Planck Equation*

It is interesting to see how this work makes contact with the work of E. Nelson (1966, 1967). Because of the result (5.24), equations (5.1) and (5.6) can be combined by addition or subtraction to give the two Fokker–Planck equations

$$\frac{\partial\rho}{\partial t} = -\nabla.[\langle \mathbf{v}_1 + \mathbf{v}_2\rangle \rho] + D\nabla^2 \rho \qquad (7.1)$$

and

$$\frac{\partial\rho}{\partial t} = -\nabla.[\langle \mathbf{v}_1 - \mathbf{v}_2\rangle \rho] - D\nabla^2 \rho \qquad (7.2)$$

on which Nelson bases his derivation of the Schrödinger equation. However, it should be stressed that the work in this paper has a physically and statistically very different structure from the structure on which Nelson bases his work. To this author it seems that two actually distinct fluids both involving positive and negative mass contributions are needed if classical fluid sense is to be seen as underlying the quantum process (Gilson, 1968a).

8. *Conclusions*

The perfection with which the group of ideas contained essentially in the papers (Gilson, 1969b, c, 1971a; Hörnqvist, 1969; Nelson, 1966, 1967) and in the present paper have here been shown to fit together within the context of this two fluid scheme would seem strongly to imply its general correctness. It is significant that the argument used to construct a mathematical model for a quantum simulating fluid complex given in this paper does not logically depend on using information derived from orthodox quantum theory, except in the minor respect of identifying the values of the viscosity coefficient ν and the diffusion coefficient D. It is remarkable that even the correct form for the temperature which involves the strange $\nabla^2 \ln\rho$ function arises naturally from the viscous fluid basis. The main conclusion to emerge from this work can be summarised as follows. Quantum processes can be described by a fluid complex formed from a polarisation of virtual positive and negative vacuum mass states by an internal thermal energisation. Material related to the ideas contained in this paper can also be found in the works of Bartlett (1949), Gilson (1968b, c, d, 1969e, d), Kershaw (1964), Leaf (1968), London (1954), Madelung (1926), Moyal (1949), Suppes (1963), Vigier (1954) and Wigner (1932).

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